Improving the mass determination of Galactic Cepheids

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ABSTRACT

We have selected a sample of Galactic Cepheids for which accurate estimates of radii, distances, and photometric parameters are available. The comparison between their pulsation masses, based on new Period-Mass-Radius (PMR) relations, and their evolutionary masses, based on both optical and NIR Color-Magnitude (CM) diagrams, suggests that pulsation masses are on average of the order of 10% smaller than the evolutionary masses. Current pulsation masses show, at fixed radius, a strongly reduced dispersion when compared with values published in literature. The increased precision in the pulsation masses is due to the fact that our predicted PMR relations based on nonlinear, convective Cepheid models present smaller standard deviations than PMR relations based on linear models. At the same time, the empirical radii of our Cepheid sample are typically accurate at the 5% level.

Our evolutionary mass determinations are based on stellar models constructed by neglecting the effect of mass-loss during the He burning phase. Therefore, the difference between pulsation and evolutionary masses could be intrinsic and does not necessarily imply a problem with either evolutionary and/or nonlinear pulsation models. The marginal evidence of a trend in the difference between evolutionary and pulsation masses when moving from short to long-period Cepheids is also briefly discussed. The main finding of our investigation is that the long-standing Cepheid mass discrepancy seems now resolved at the 10% level either if account for canonical or mild convective core overshooting evolutionary models.

Subject headings: stars: Cepheids – stars: evolution – stars: fundamental parameters – stars: oscillations

1. Introduction

Classical Cepheids are important objects not only because they are excellent distance indicators but also because it is possible, on the basis of both direct and indirect methods, to evaluate their intrinsic parameters such as radii, masses, and effective temperatures. A large amount of theoretical and empirical investigations has been devoted to classical Cepheids and the empirical uncertainties affecting their intrinsic parameters are by far smaller than for any other group of variable stars. As a consequence, Cepheids are key objects to constrain theoretical predictions.

According to the so-called pulsation relation the period of a variable depends, at fixed chemical composition, on stellar mass, luminosity (radius), and effective temperature (Bono, Castellani, Marconi 2000a). If we neglect the width in temperature of the instability strip the pulsation relation becomes a Period-Mass-Radius (PMR) relation. This means that independent estimates of both period and mean radius supply an independent evaluation of Cepheid masses (called pulsation masses). Theoretical PMR relations based on nonlinear (Christy 1968; Fricke, Stobie, & Strittmatter 1972, hereinafter FSS72) pulsation models have been widely adopted in the literature to estimate the Cepheid pulsation masses (Gieren 1989, hereinafter G89; Nordgren et al. 2000, hereinafter N00). On the other hand, Cepheid masses can be estimated on the basis of the Mass-Luminosity (ML) relation for intermediate-mass stars predicted by evolutionary models (Becker, Iben, & Tuggle 1977; Stothers & Chin 1994; Bono et al. 2000b; Baraffe & Alibert 2001). This method requires a determination of the Cepheid distance and reddening as well as the use of a relation between period and bolometric correction (BC, as given in G89). A slightly different approach for deriving Cepheid evolutionary and pulsation masses has been recently devised by Beaulieu, Buchler, & Kollath (2001, hereinafter BBK01). They derived, by adopting the Kurucz (1995) stellar atmosphere models, new analytical relations for both effective temperatures and BCs as a function of two observables, namely the period and the mean color (V-I).

The comparison between pulsation and evolutionary masses led to the problem of Cepheid mass discrepancy (Cox 1980). Early estimates suggested that pulsation masses were approximately a factor of two smaller than the evolutionary masses. This problem has been substantially alleviated by the inclusion in the pulsation codes of the new OPAL (Rogers & Iglesias 1992; Iglesias & Rogers 1996) and OP (Seaton et al. 1994) radiative opacities (Moskalik et al. 1992; Kanbur & Simon 1994). Despite the improvement in the accuracy of radiative opacities the agreement is far from being satisfactory, and indeed several doubts were raised for Magellanic Cepheids by Buchler et al. (1996) and by Wood, Arnold, & Sebo (1997). Progress was made by Bono, Marconi & Stellingwerf (2000c) for the group of Cepheids that show a bump along the light curve (Bump Cepheids). They

obtained, on the basis of a fine grid of nonlinear, convective models relevant for Bump Cepheids ($6.9 \leq P \leq 17.8$ days), a reasonable agreement between predicted and empirical masses. We have now to extend this work to the full period range covered by classical Cepheids, which is one of the main goals of this investigation. At the same time, we are also interested in checking the suggestion of BBK01, based on the OGLE database for Magellanic Cepheids, that current ML relations lead to evolutionary masses that are systematically larger by ≈ 0.1 dex in $M/M_{\odot}$ than the masses predicted by pulsation models.

In §2 we discuss the theoretical framework and the selection of the Cepheid sample adopted in this investigation. In §3 we supply suitable analytical relations for fundamental (F) and first overtone (FO) pulsators connecting the stellar mass to periods, and radii. In this section we also estimate pulsation and evolutionary masses. Finally, in §4 we compare Cepheid masses based on pulsation and evolutionary predictions and provide plausible hypotheses to account for their difference.

2. Theoretical models and empirical data

Even though current spectroscopic measurements seem to suggest that the metallicity of Galactic Cepheids ranges from Z=0.008 to Z=0.024 (Fry & Carney 1997), we adopted the chemical composition typical of solar neighborhood Cepheids, i.e. Y=0.28, Z=0.02. To supply homogeneous predictions for both F and FO PMR relations we used the same theoretical framework adopted by Bono, Caputo, & Marconi (1998), Bono, Marconi, & Stellingwerf (1999, hereinafter BMS99) and by Bono et al. (2001, hereinafter BGMF01). To account for the current uncertainty in the predicted luminosity of intermediate-mass stars, we derived two PMR relations for canonical and noncanonical Cepheid models. These models were constructed by adopting two different ML relations based on evolutionary models that neglect (canonical, BMS99; Bono et al. 2000a) or include a mild convective core-overshooting (noncanonical, Girardi et al. 2000).

The grid of canonical and noncanonical fundamental models constructed by BMS99 for stellar masses ranging from 5 to 11 $M_{\odot}$, were implemented with the new sequences of canonical models for $M/M_{\odot}$ =4.5, 6.25, 6.5, 6.75 computed by BGMF01. We also constructed a new sequence of F models ($M/M_{\odot}$ =4.0, log $L/L_{\odot}$ =2.97) to properly cover the short-period range of noncanonical pulsators. As far as the first overtone is concerned, we adopted the canonical ($3.5 \leq M/M_{\odot} \leq 5.5$) and the noncanonical ($3.0 \leq M/M_{\odot} \leq 4.75$) models provided by BGMF01. These models cover a large fraction of the Galactic Cepheid instability strip, and indeed the periods range from 2.7 to 132.4 days for F and from 1.3 to 3.8 days for FO pulsators. Table 1 lists the coefficients and the relative errors of the

analytical PMR relations for the two modes we studied. Note that these relations were derived by adopting nonlinear, convective models that show a stable limit cycle.

To test the accuracy and plausibility of current models we also derived the PMR relations for fundamental pulsators by adopting as independent variables the stellar mass and the radius. We found:

$$\log P(C) = -1.70(\pm 0.01) - 0.90(\pm 0.12) \log M + 1.86(\pm 0.04) \log R \qquad \sigma = 0.01$$

$$\log P(NC) = -1.74(\pm 0.02) - 0.70(\pm 0.15) \log M + 1.80(\pm 0.06) \log R \qquad \sigma = 0.01$$

where the symbols have their usual meaning. These relations suggest that the periods of fundamental Cepheids do not scale according to the period-density relation i.e. $P \propto R^{1.5}/M^{0.5}$, but as $P \propto R^{1.8}/M^{0.8}$. However, the fundamental period of convective oscillating stellar envelopes is proportional to R^2/M as originally demonstrated by Gough, Ostriker, & Stobie (1965) on the basis of polytropic models. The coefficients of current fundamental PMR relations are in good agreement with these leading physical arguments. Note that the difference between the periods predicted by the period-density relation and by nonlinear PMR relations is vanishing for periods shorter than 10 days but becomes of the order of 25-30% for periods equal to 60 days.

In order to provide a detailed comparison between theory and observations we selected the sample of radii, and distances determined by Gieren, Fouqué, & Gomez (1997,1998, hereinafter GFG97 and GFG98) for 34 Galactic Cepheids. The reason why we selected this particular sample is twofold: i) radii and distances were derived by adopting for the entire sample the IR Barnes-Evans surface brightness technique (Fouqué & Gieren 1997). This approach presents several indisputable advantages when compared with other methods available in the literature and supply homogeneous and accurate evaluations of radii and distances (GFG97). ii) for these objects both optical and NIR absolute magnitudes are available, as well as individual reddening estimates. We excluded from this sample the three longest period Cepheids (SV Vul, GY Sge, and S Vul) for the reasons already outlined by GFG98, and CS Vel since for this object no I band magnitude is available. However we included Polaris, since for this object an independent good estimate of the radius is available (N00). As a result, we end up with a sample of 31 Cepheids, which form the database for our current comparison between theory and observations.

3. Pulsation and evolutionary masses

Fig. 1 shows the pulsation masses for the Galactic Cepheids in our sample as a function of the radius. The new masses were estimated by adopting the PMR relations listed in Table 1 and present a smaller dispersion when compared with similar estimates available in the literature (e.g. G89). In particular, we found that pulsation masses estimated by adopting the PMR relations derived by FSS72 and Li & Huang (1990) attain similar values but the intrinsic dispersion is at least a factor of two larger when compared with our estimates. The reason for this improvement is threefold: i) previous relations rely on old input physics (opacities, and equation of state) and cover a narrower mass range; ii) the previous PMR relations are based on nonlinear, radiative models (FSS72), and linear, nonadiabatic models (Li & Huang 1990), therefore they do not supply reliable estimates of the location of the red edge; iii) the small dispersion of pulsation masses is also due to the dramatic improvement in the accuracy of empirical radius measurements (the error bars are quite often smaller than the size of the symbols), when compared with former estimates (Cogan 1978; G89).

As expected the Cepheid masses predicted by noncanonical PMR relations (triangles) are systematically smaller when compared with the canonical ones (filled circles). In fact, the He core masses of noncanonical models are, at fixed luminosity and chemical composition, larger than for canonical models. Data plotted in this figure present mild evidence that such a difference increases toward shorter periods. This effect could be due to the stronger sensitivity of the fundamental instability strip edges to stellar mass toward lower luminosities (Bono, Caputo, & Marconi 2001). According to a recent detailed analysis of short-period Cepheids, Polaris (square) and SZ Tau (diamond) are almost certainly first overtone pulsators (N00; BGMF01). Unfortunately, the current uncertainties on the radius of SZ Tau does not allow us to constrain its pulsation mass. However, for Polaris we found that the pulsation mass is of the order of $M_p/M_\odot = 4.9 \pm 0.1$. This error just accounts for the uncertainty due to the use of canonical and noncanonical PMR relations. This estimate is in very good agreement with the pulsation mass obtained by N00, i.e. $M_p/M_\odot = 4.5 \pm 2$. The large difference in the error is due to the fact that we adopted PMR relations for FO pulsators, whereas the former authors were forced to fundamentalize the period of Polaris and then to use PMR relations for fundamental pulsators. In fact, up to now a PMR relation for FO Cepheids was not available. Note that the intrinsic dispersion of FO PMR relations is almost a factor of three smaller than for F pulsators (see last column in Table 1). As already noted for the Period-Radius relation (BGMF01) this difference is due to a decrease in the width in temperature of the instability strip and suggests that FO masses are practically unaffected by the intrinsic spread of the instability strip.

In order to estimate the evolutionary masses we devised the following approach. We first plotted our Cepheid sample in the Color-Magnitude diagram (CMD), since it has been suggested that current evolutionary tracks are at odds with Magellanic short-period Cepheids (BBK01). To account for subtle errors both in the distance, as well as in the reddening correction we adopted an optical (M_V, V-I) and a NIR (M_K, J-K) CMD (see Fig. 2). Evolutionary tracks for intermediate-mass stars ($3 \leq M/M_{\odot} \leq 14$) recently computed by Bono et al. (2000b) for Y=0.29[1] and Z=0.02 were adopted to construct isochrones ranging from 10 to 200 Myr. Theoretical predictions were transformed into the observational plane by adopting the bolometric corrections and the color-temperature relations by Bessell, Castelli, & Plez (1998). A glance at the data plotted in the two panels shows that the empirical data are in very good agreement with stellar isochrones (see labeled ages). In particular, we find that predicted blue loops account for the distribution of observed Cepheids even at short periods.

The evolutionary masses were estimated by adopting a detailed set of stellar isochrones, with an age step of 2 Myr from 11 to 35 Myr, of 5 Myr from 35 to 100 Myr, and of 10 Myr from 100 to 150 Myr. For each object the mass was estimated by performing an average over the masses that fall inside the empirical error box, i.e. the box given by the errors on distance and colors. Fig. 3 shows evolutionary mass estimates based on both the (V, V-I) and the (K, J-K) CMDs as a function of period. The most important conclusion from this figure is that Cepheid masses based on optical and NIR CMDs agree at the level of few percent and are consistent within their uncertainties. However, evolutionary masses based on optical magnitudes seem to be slightly systematically larger than their NIR counterparts. This could be due to an error in the zero-point of the adopted reddening scale, since NIR magnitudes are practically unaffected by reddening corrections in contrast with optical magnitudes. The star-to-star differences between Cepheid masses based on optical and NIR magnitudes could be due to variable individual errors in the reddening correction to the V magnitudes. Moreover, it is worth mentioning that K magnitudes will be only marginally affected by the presence of an unresolved companion (physical or photometric blend), since Cepheid companions are expected to be on average hotter objects. This is not true for the V magnitudes that can be substantially affected by the luminosity of the blue companion. This effect could also contribute to the variable mass difference for individual Cepheids which is observed in Fig. 3.

[1]Note that the marginal difference in the He content adopted for constructing the pulsation models (0.28 against 0.29) has negligible effects on current conclusions.

4. Discussion and conclusions

In order to perform a quantitative comparison between evolutionary and pulsational masses we performed a linear fit through current optical and NIR evolutionary masses and we found:

$$\log M_e/M_\odot = -0.03(\pm 0.02) + 0.48(\pm 0.01) \log R/R_\odot \qquad \sigma = 0.02$$

where σ is the standard deviation. The same fit performed over the pulsation masses based on the canonical ML relation yields:

$$\log M_p/M_\odot = -0.09(\pm 0.03) + 0.48(\pm 0.03) \log R/R_\odot \qquad \sigma = 0.03.$$

The two fits yield, within the small error, identical slopes; however, they show that evolutionary masses are systematically larger when compared with the pulsation ones. The difference ranges from $\approx$0.05 dex at $\log R/R_\odot$=1.6 to $\approx$0.04 at $\log R/R_\odot$=2.2. Taken at face value this difference implies a discrepancy between evolutionary and pulsation masses of the order of 13% for short-period Cepheids and of the order of 10% for long-period ones. Within current empirical uncertainties we cannot assess whether this trend is real or caused by the small number of long-period Cepheids included in our sample. The same result is demonstrated in yet another way i.e. by plotting the Cepheid masses as function of period. Fig. 4 shows the ratio between pulsation and evolutionary masses versus period. The pulsation masses are the canonical ones, while the evolutionary masses are a mean between the mass evaluations based on optical and NIR magnitudes. The pulsation masses are systematically smaller than the evolutionary masses by $\approx$ 10% and the spread is of the order of 10%. Data plotted in Fig. 4 should be compared with Fig. 5 in G89 to appreciate the improvement in the accuracy of current mass determinations. The sample of Galactic Cepheids we are dealing with is too small to reach any firm conclusion on the occurrence of a trend with the pulsation period.

However, in this context it is worth mentioning that evolutionary masses are based on stellar models that neglect mass-loss during He-burning phases, whereas the pulsation masses do estimate the actual mass of Cepheids. Therefore, the mass difference we obtain could be a real feature, reflecting a mass loss of Cepheids of the order of 10% during the He burning phase, and not the consequence of errors in the predictions based on evolutionary and/or pulsation models. As our most important conclusion, we stress that the long-standing discrepancy between evolutionary and pulsation masses has been brought down, for the first time, to the 10% level. Moreover, we point out that the current small

residual discrepancy between evolutionary and pulsation masses is not affected by the adopted evolutionary scenario. In fact, pulsation masses based on noncanonical PMR relations are smaller when compared with the canonical ones, but the evolutionary masses based on noncanonical evolutionary tracks are smaller as well. Thus the ratio between the two mass estimates does not change. The comparison with the G89 results discloses that we have gone a long way since then in improving Cepheid models and empirical determinations of Cepheid radii and distances.

Regarding the possibility of mass loss we are not aware of any recent spectroscopic investigation aimed at measuring the efficiency of mass-loss among both Galactic and Magellanic Cepheids. Empirical estimates based on infrared (IRAS) and ultraviolet (IUE spectra) emissions for a sizable sample of Galactic Cepheids suggest mass loss rates ranging from 10^{-10} to 10^{-7} $M_\odot\, yr^{-1}$ (Deasy 1988). However, observational determinations are not well constrained, and indeed VLA observations (Welch & Duric 1988) and resonance absorption line profiles (Rodrigues & Böhm-Vitense 1992) provide upper limits ranging from 10^{-10} to 10^{-7} $M_\odot\, yr^{-1}$. In spite of the empirical uncertainties these values seem to support our hypothesis that the discrepancy between evolutionary and pulsation masses could be due to mass loss. In fact, the He burning lifetimes for a 5 $M_\odot$ Cepheid at solar chemical composition is of the order of 20 Myr while for a 11 $M_\odot$ Cepheid it is 2.5 Myr. The lack of a firm empirical evidence concerning the correlation of the mass loss efficiency with the pulsation period limits the significance of the result. Future theoretical and empirical investigations to shed new light on Cepheid mass loss will be of great value.

Current results seem to suggest that the discrepancy between evolutionary and pulsation masses for Galactic Cepheids is roughly a factor of two smaller (0.05 against 0.1 dex) than found by BBK01 for Magellanic Cepheids. Since, we adopted a different approach to estimate both evolutionary and pulsation masses, we cannot assess whether this difference is due to an intrinsic feature of Galactic Cepheids or to unknown systematic errors. Note that our pulsation masses based on the PMR relations do rely on periods and mean radii, whereas the approach suggested by BBK01 requires periods, mean magnitudes, colors, and reddening corrections. Therefore our approach seems to be more straightforward and less vulnerable to errors in the adopted empirical data. On the other hand, evolutionary masses were estimated by comparing theory and observations directly into the CMDs, whereas BBK01 adopted different ML relations. New empirical estimates of Magellanic Cepheid radii and distances based on the infrared Barnes-Evans surface brightness technique will be crucial to address in the near future this apparent discrepancy between our results and those of BBK01.

In passing we note that the evolutionary mass of SZ Tau is in good agreement with

the Cepheid masses of short-period Cepheids. This suggests that the distance modulus is not affected by large uncertainties while the empirical radius, and in turn the pulsation mass, are affected by larger errors. Even though the distance of Polaris has been recently estimated by Hipparcos (Feast & Catchpole 1997) we cannot estimate its evolutionary mass, since I, J, and K magnitudes are not available for this object.

It is a pleasure to thank S. Cassisi for sending us the detailed set of isochrones at solar chemical composition and for several thorough discussions on the mass loss efficiency among intermediate-mass stars. We also wish to thank an anonymous referee for his/her pertinent suggestions that improved the readability of the paper. GB & MM acknowledge financial support by MURST-Cofin 2000, under the scientific project "Stellar observables of cosmological relevance". WPG gratefully acknowledges partial financial support received from Fondecyt projects 1000330 and 8000002. Part of this work was carried out while WPG was a scientific visitor at the European Southern Observatory in Garching. WPG is grateful for the support provided by ESO. He would also like to acknowledge computer support from the Center for International Migration (CIM) in Germany.

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TABLE 1. PMR relations at solar chemical composition: $\log M = \alpha + \beta \log P + \gamma \log R^a$

	α	β	γ	σ
		Fundamental		
C[b]	-0.966 ± 0.012	-0.599 ± 0.078	1.291 ± 0.118	0.011
NC[b]	-1.073 ± 0.015	-0.608 ± 0.133	1.334 ± 0.200	0.014
		First Overtone		
C[b]	-2.776 ± 0.004	-1.661 ± 0.140	2.682 ± 0.185	0.004
NC[b]	-2.034 ± 0.004	-1.179 ± 0.246	2.068 ± 0.334	0.004

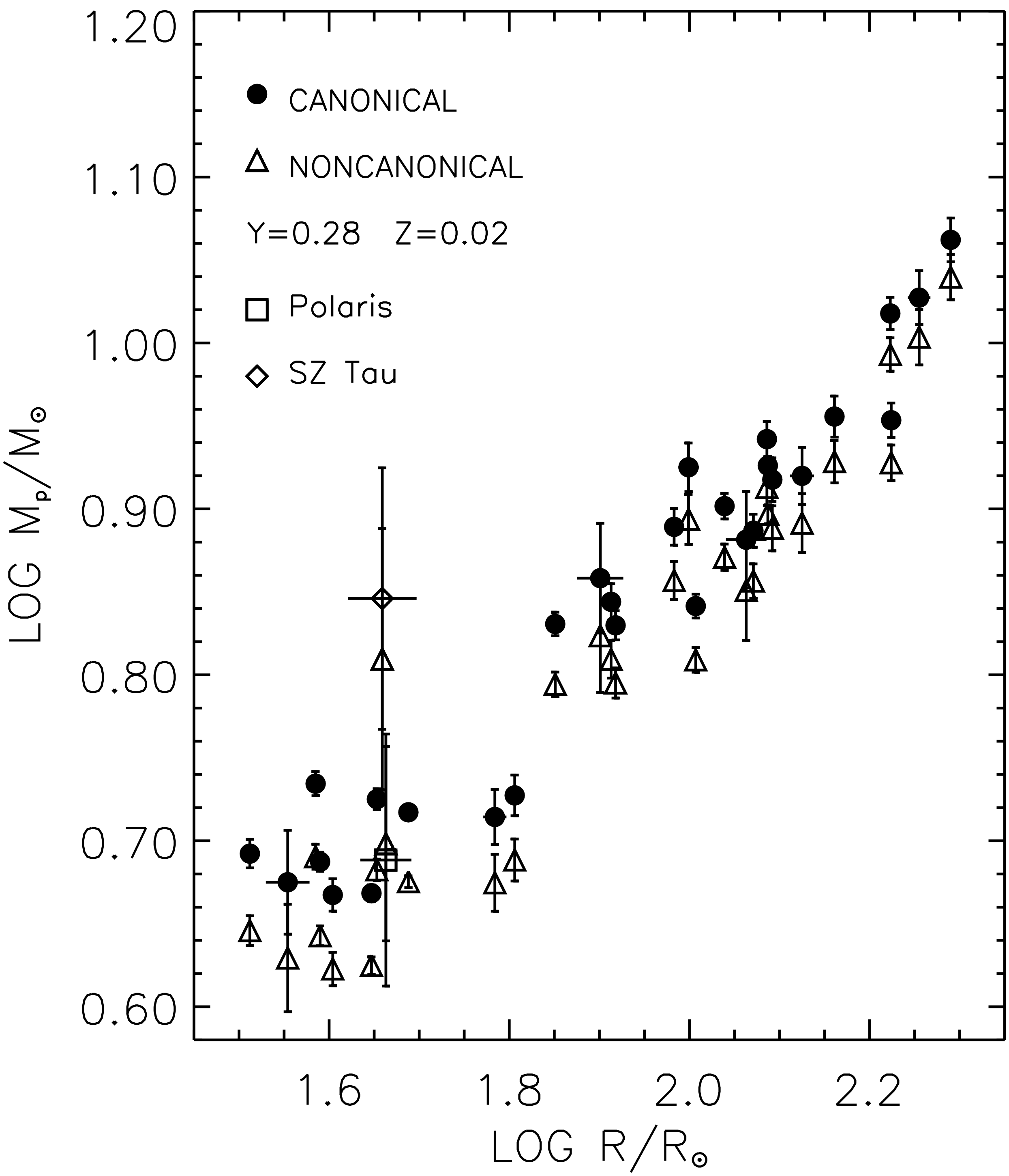
[a] Stellar masses and radii are in solar units, while the periods are in days. [b] PMR relations based on Cepheid models constructed by adopting canonical (C) and noncanonical (NC) ML relations.

Fig. 1.— Pulsation masses as a function of radius for our Cepheid sample (see text). Solid circles and triangles refer to mass estimates of fundamental mode variables based on canonical and noncanonical ML relations respectively. The square and the diamond mark two first overtones, namely Polaris and SZ Tau. The error bars on the mass account for the uncertainty in the coefficients of the PMR relations. The error bars in the mean radius are typically smaller than the size of the symbol.

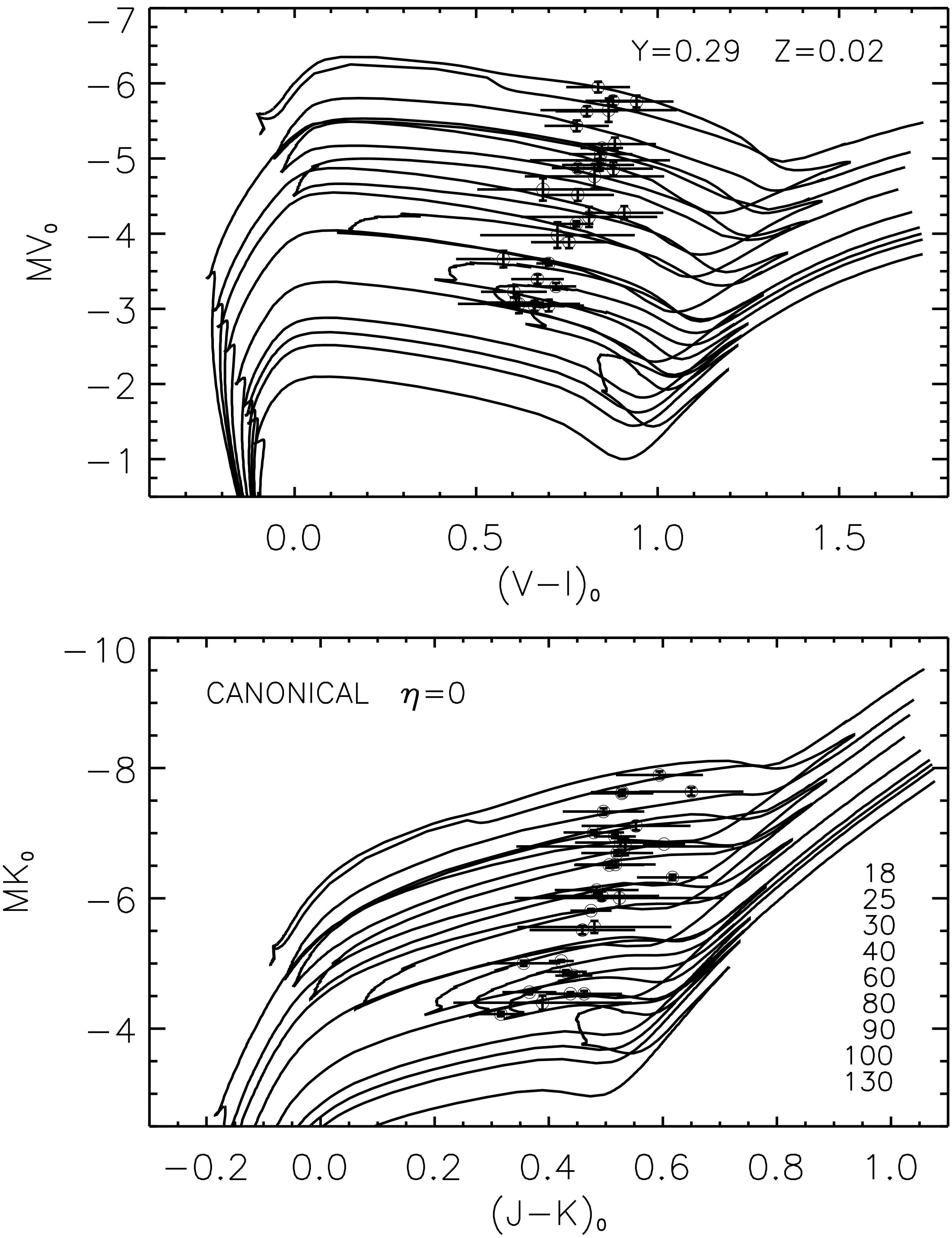
Fig. 2.— Top: Color-Magnitude diagram (M_V, $(V-I)_0$) for our sample of Galactic Cepheids. Solid lines show theoretical isochrones at solar chemical composition for stellar ages ranging from 18 to 130 Myr. The error bars account for the uncertainty both in the distance modulus and in the reddening correction. Bottom: same as the top panel but for NIR bands (M_K, $(J-K)_0$). The ages of individual isochrones are also labeled.

Fig. 3.— Evolutionary masses as a function of radius for our sample. Solid circles and triangles refer to mass estimates based on the optical and NIR CMDs, respectively. The evolutionary masses were estimated by adopting stellar isochrones, and in turn evolutionary tracks that neglect mass loss ($\eta = 0$) and convective core overshooting (canonical). The solid line shows the linear fit between empirical radii and Cepheid masses.

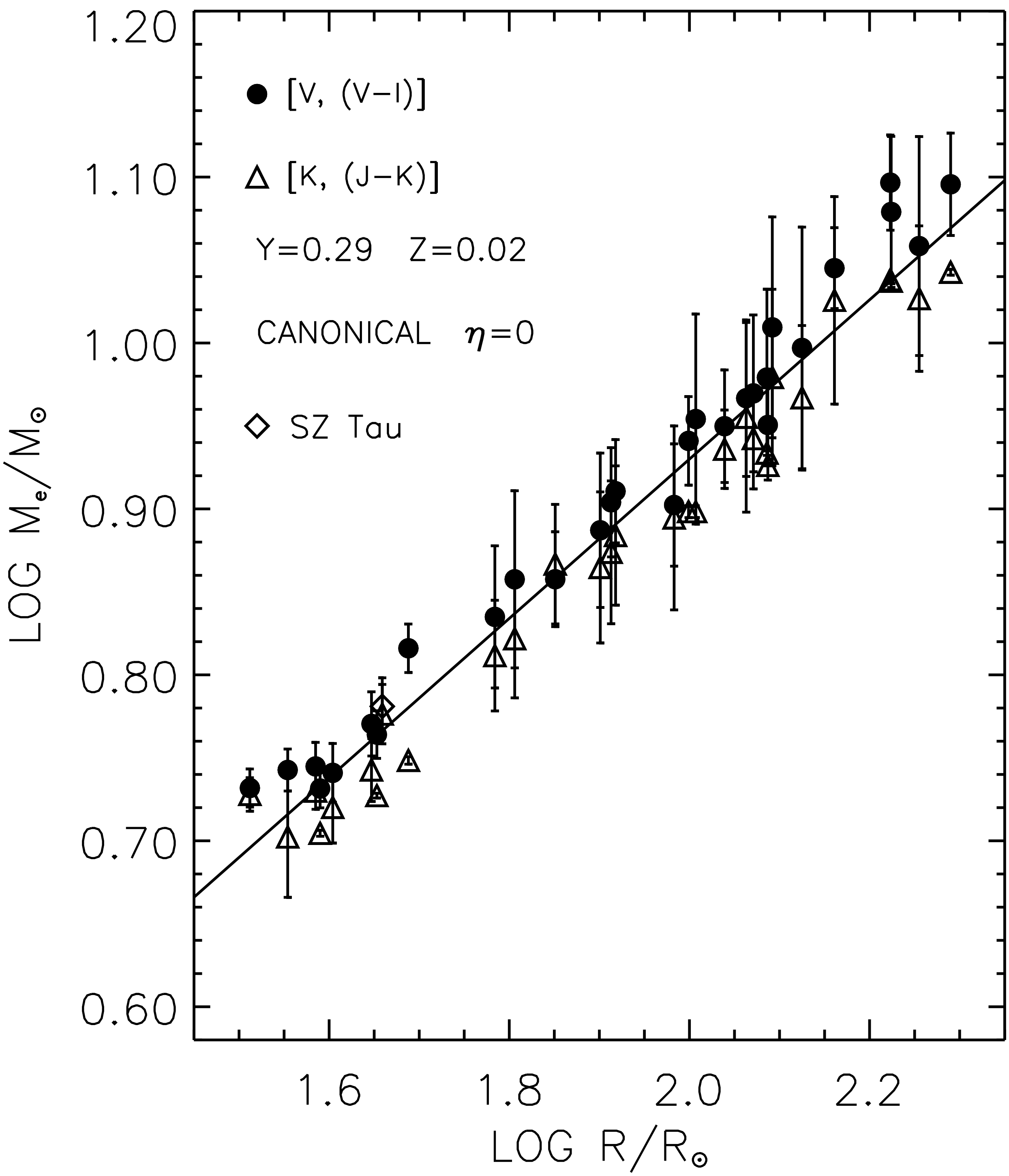
Fig. 4.— Ratio between pulsation and evolutionary masses versus logarithmic period for the fundamental Cepheids in our sample. Pulsation masses were estimated by adopting the canonical PMR relation, while the evolutionary ones are the mean between the mass estimates based on optical and NIR magnitudes.



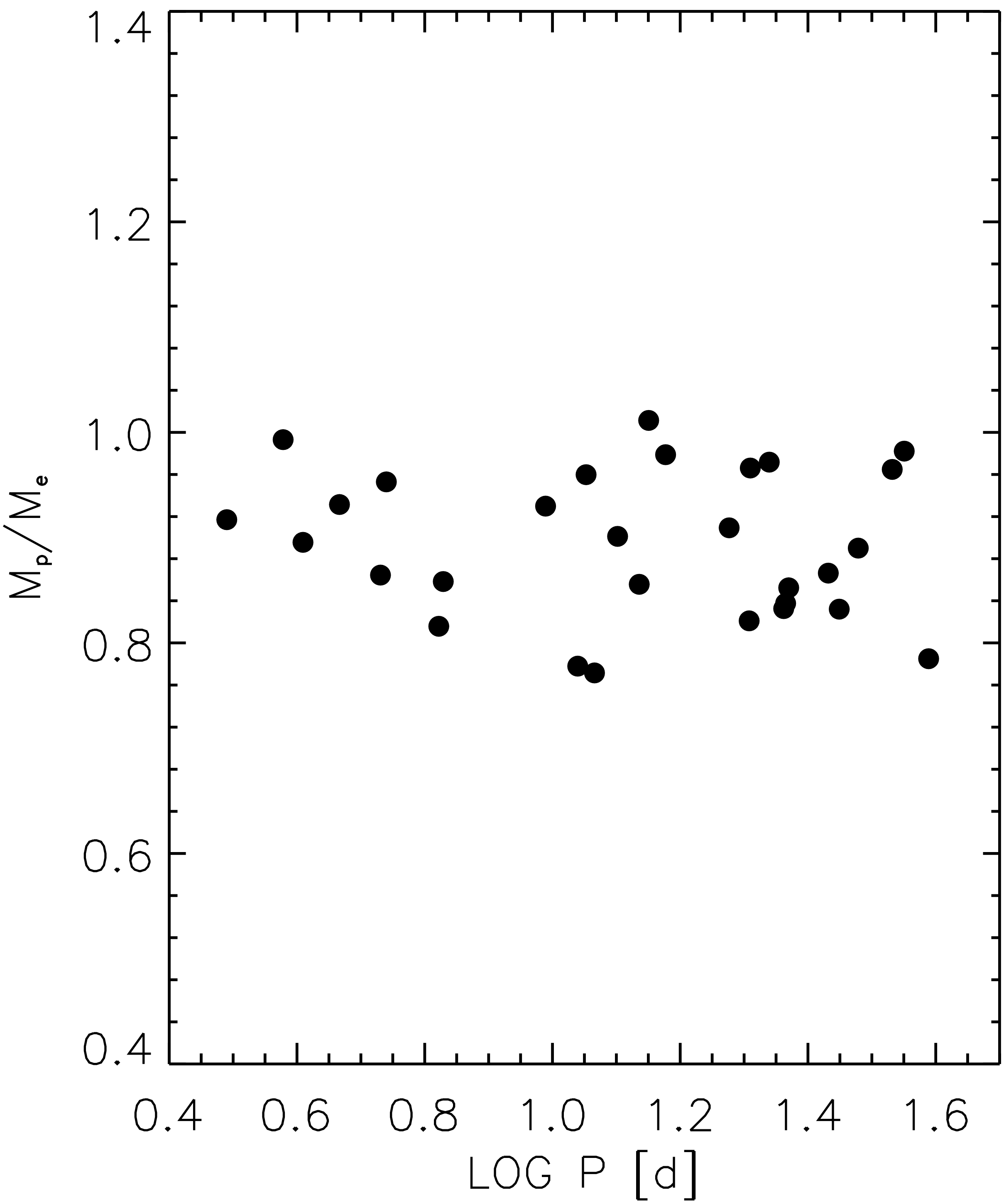
CANONICAL
NONCANONICAL
Y=0.28 Z=0.02
Polaris
SZ Tau
1.20
1.10
1.00
0.90
0.80
0.70
0.60
LOG $M_p/M_\odot$
1.6
1.8
2.0
2.2
LOG $R/R_\odot$



Y=0.29 Z=0.02
MV₀
(V−I)₀
CANONICAL η=0
MK₀
(J−K)₀
18
25
30
40
60
80
90
100
130



1.20
1.10
1.00
0.90
0.80
0.70
0.60
LOG $M_e/M_\odot$
[V, (V−I)]
[K, (J−K)]
Y=0.29 Z=0.02
CANONICAL η=0
SZ Tau
1.6
1.8
2.0
2.2
LOG $R/R_\odot$



1.4
1.2
1.0
0.8
0.6
0.4
M_p/M_e
0.4
0.6
0.8
1.0
1.2
1.4
1.6
LOG P [d]